COMPRESSION AND BUCKLING OF RODS IN CREEP UNDER CONSTANT LOADS AND AT MONOTONICALLY INCREASING TEMPERATURES

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An investigation of compression and buckling of rods at constant temperatures and under monotonically increasing axial loads was described in [1]. This article describes the continuation of work concerned with another set of extreme conditions, i.e., a constant load and temperature monotonically increasing at a certain given rate. Experimental results are reported and certain possible ways of describing them by mathematical expressions are discussed.

§1. Using a method described in [1], we carried out a series of compressive creep tests on material D16T under monotonically increasing loads at constant temperatures ranging from 250 to 400° C (inclusive) at 25° C intervals. For each test temperature the relation between stress and total strain at a given loading rate was plotted.

By assuming that total strain ε is a sum of instantaneous elastic-plastic strain ε and creep strain p,

$$\epsilon = \varepsilon + p, \tag{1.1}$$

and by taking into account the fact that at $T \geq 275°$ C creep of the material in question takes place without strain-hardening, we estimated the last term in (1.1) from

$$dp = K e^{\beta\sigma}\, dt. \tag{1.2}$$

For creep under loads monotonically increasing at a constant load

$$\sigma = ct. \tag{1.3}$$

After substituting into (1.2) and integrating for such starting conditions, we obtain

$$p = k\,(e^{\beta\sigma} - 1)\,/\,\beta c. \tag{1.4}$$

Usually $e^{\beta\sigma} \gg 1$; if this is taken into account and if test results for each temperature are plotted in log p and σ coordinates, it is possible to determine K and β.

In Fig. 1 results of tests at 325° C are plotted in this way; curves 1, 2, and 3 relate to data obtained at loading rates $c_1 = 0.18$, $c_2 = 0.01$, and c_3 kgf/mm²/sec.

Experimental points lie on lines which are almost straight, though there is a tendency for their slope (and, consequently, the magnitude of β) to increase [1]. Reducing loading rate leads to a certain increase in β. The latter effect is evidently associated with aging phenomena in alloy D16T: the slower the loading rate, i.e., the longer the time during which the alloy is held at a given temperature, the more substantial is the reduction in the alloy strength [2]. As a result, one can talk only about certain average values K and β for a given temperature and stress interval. Similar graphs were obtained for other test temperatures.

Values K and β determined by the above described method monotonically increase with rising temperature; their numerical values can be satisfactorily approximated with the aid of the Lagrange interpolation polynomial.

Confining ourselves to third-power polynomials, we obtain for K

$$K = (3.488 \cdot 10^{-6} T^3 - 25.524 \cdot 10^{-4} T^2 + 62.065 \cdot 10^{-2} T - 50.101) \cdot 10^{-8}\ \mathrm{sec}^{-1}. \tag{1.5}$$

Analogically for β

$$\beta = 0.5066 \cdot 10^{-6} T^3 - 4.1994 \cdot 10^{-4} T^2 + 12.0916 \cdot 10^{-2} T - 11.2983\ \ [\mathrm{mm^2/kG}], \tag{1.6}$$

where T appears in °C.

Thus, (1.12), in the entire temperature interval 250–400° C, becomes

$$dp = K(T)\exp[\beta(T)\sigma]\,dt. \tag{1.7}$$

Figure 2 shows graphs σ-E plotted for rods tested in compression at a) 275, b) 325, c) 400° C; numbers 1, 2, 3, and 4 ascribed to experimental points indicate, respectively, results obtained at loading rates of $c_1 = 4.5$, $c_2 = 0.18$, $c_3 = 0.01$, $c_4 = 0.0045$ kG/mm² sec.

Theoretical curves obtained from (1.7) and (1.1) are shown as dashed lines. Similar results were obtained for other test temperatures. It can be stated in conclusion that creep at constant temperatures is satisfactorily described by (1.7) in the entire temperature interval studied.

Validity of (1.7) with functions K = K(T) and β = β(T) of type (1.5) and (1.6), respectively, was verified on experimental data in creep of the same material at constant loads and temperatures monotonically increasing at a constant rate.

The test pieces were prepared from alloy D16T rods in the form of cylinders 70 mm long with the gauge portion 40 ± 0.1 mm long and 7.5 ± 0.01 mm diameter. To ensure more uniform heating under non-steady-state conditions, tube specimens (internal diameter = 4 ± 0.01 mm) were used. At slow heating rates both tube and solid specimens of equal cross section were used; the test results were identical.

The specimens were heated at linearly increasing rates with the aid of an "EKVT" type self-recording instrument with a temperature controlling device. The specimen temperature was controlled by closing and opening contacts in the furnace heater circuit. The master switch is connected by a cable to the drum of the self-recorder tape winder and moves at a constant speed in the direction of increasing temperature scale. The other contact in the form of flat spring is attached to the carriage of the self-recorder stylus which records the specimen temperature. When in the working position, the stylus carriage with its contact always leads the master switch. The latter, moving at a constant speed, catches up with the carriage and closes the heater circuit; as a result, the furnance temperatures rises and the carriage, following the specimen temperature, moves ahead of the master switch thereby opening the heater circuit.

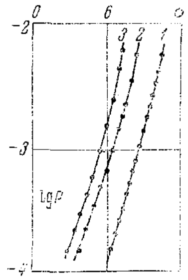

Fig. 1

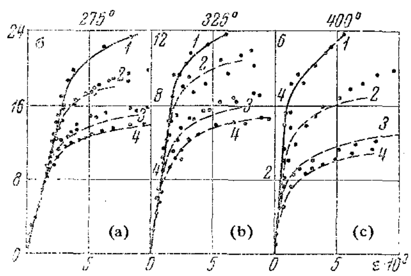

Fig. 2

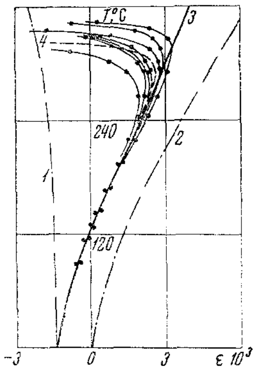

Fig. 3

When the temperature drops, the carriage stops and starts moving in the opposite direction; the master switch catches up again with the carriage and the process is repeated. As a result, the specimen temperature oscillates about the preset values within ±3° C with corresponds to ±1.0% of the median temperature. The readings of three chromel-copel thermocouples (contact-welded to the specimen) were recorded by an "EKVT" type compensating self-recorder.

Tests were carried out at three heating rates: θ = 1.66° C/sec; θ_2 = 0.52° C/sec. The axial strain of each specimen was recorded during the tests.

A series of experiments was carried out to determine thermal strains at different heating rates. It was established that heating rate (in the range studied) has no effect on the magnitude of thermal strains. At heating rates faster than θ_1 = 1.66° C/sec strains were smaller than those measured at the corresponding temperature and slower heating rates. This effect is evidently attributable to nonuniform heating at fast heating rates.

As pointed out above, alloy D16T age-hardens at elevated temperatures. All experiments at constant temperatures, from the results of which (1.5) and (1.6) were determined, were carried out in the following way: heating to a given test temperature, holding the specimen at the temperature for 20 min, and programmed loading. To obtain comparable results and to make it possible to us (1.5) and (1.6), experiments at increasing temperatures were carried out in the following way: heating to 300° C, holding at the temperature for 30 min, cooling to room temperature, applying a constant load, and switching on the programmed heating. Tests at above cited heating rates were carried out at two constant loads: σ_1 = 8.1 and σ_2 = 10 kG/mm².

Results of tests at θ = 0.52° C/sec and σ = 8.1 kgf/mm² are reproduced in Fig. 3. Curve 1 represents the diagram of elastic strains $\varepsilon_1 = \sigma/E(T)$ which increase due to the fact that the elasticity modulus decreases with temperature. Curve 2 represents the diagram of thermal strains $\varepsilon_2 = \alpha T$. Curve 3 represents the variation in the sum $\varepsilon_1 + \varepsilon_2$.

Experimental points which represent the sum of thermal, elastic and creep strains are shown as black dots. As was to be expected, up to temperatures of about 200° C the experimental points coincide with curve 3; this indicates the absence of creep strains. At higher temperatures the experimental points deviate from curve 3 by the magnitude of the total creep strain. The dashed line (curve 4) represents calculated creep strains p obtained from (1.7) for the experimental conditions used; calculations were carried out with the aid of the Simpson formula. It will be seen that the theoretical curve passes through the zone of experimental points.

For comparison, several tests were carried out with and without preliminary heating of the specimens to, and holding them for 30 mins at, 300° C. The experimental points for specimens tested without preliminary heating were above the calculated values, while those recorded for previously heated specimens were below. This fact, indicating a marked influence of aging phenomena associated with the time during which specimens dwell at elevated temperatures, makes quantitative analysis of creep at variable temperatures rather difficult, since it introduces an additional source of error which is bound to be reflected in increased scattering of experimental points. Bearing this in mind, it may be concluded that (1.7) makes it possible to describe

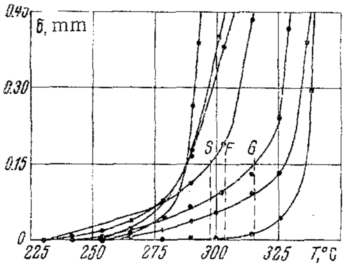

Fig. 4

creep at variable temperatures and may be used in theoretical calculations.

§2. Buckling tests were carried out on cylindrical specimens 120 mm long and 7 mm diameter. Before testing the specimens were heated to 300° C and held at the temperature for 20 min. After cooling to room temperature the specimens were checked for geometrical imperfections. The conditions of hinged support and the method of measuring reflections and critical time were described in [1]; the heating program was described above.

A large number of experiments at a heating rate of 0.52° C/sec and stresses σ_2 = 10 kgf/mm² and σ_1 = 8.1 kgf/mm² were carried out. Figure 4 shows the results of seven experiments at σ_1 = 8.1 kgf/mm² in the form of curves representing deflection dependence on temperature at a given heating rate; similar curves were obtained at σ_2 = kgf/mm².

Experimental results were compared with calculated data obtained from the same criteria used to estimate the critical stress and time for thin-walled tubes in creep that were analyzed in [1] for the case of constant temperature and increasing loads.

According to [3] it is assumed that a straight rod will start buckling after an infinitely small lateral disturbance if the following condition is satisfied:

$$\sigma = \pi^2 E_\tau / \lambda^2 \; ; \tag{2.1}$$

here λ is flexibility and E_T is the tangent modulus of "isochronous" curve σ–ε in which time is the series parameter. In the case under consideration the role of the series parameter is played by the temperature reached at a certain constant heating rate. It is easy to show that this essentially coincides with the method of "isochronous" curves. Thus, let us consider the σ–ε plane. Let a specimen loaded to a certain stress σ_1 at room temperature be heated at a constant rate. The rise in temperature is accompanied by an increase in strain consisting of three components: a) elastic strain increasing with temperature due to a reduction in the elasticity modulus; b) creep strain; c) thermal strain. By having calculated the latter (which is stress-independent), one can measure the magnitude of the remaining two components at any given stress level and plot a graph. Having done this for all the stress levels and joining points relating to the same temperatures, we obtain a series of "isotemperature" curves σ–ε reproduced in Fig. 5. Since the heating rate at all the stress levels was the same: $dT/dt = \theta$ and $T_n = \theta t_n$; "isotemperature" curves coincide (accurate to a certain factor) with "isochronous" curves.

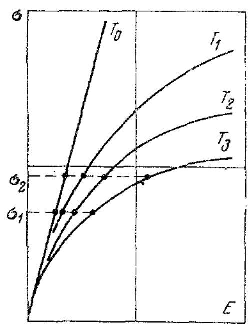

Fig. 5

Thus, the formulation of the stability problem (buckling resistance) considered in [3] can be applied without any substantial changes. By substituting in

$$1 / E_\tau = d\epsilon / d\sigma$$

the value ε from (1.1), taking into account (1.7), and bearing in mind that stresses are in the elastic range, we obtain

$$\frac{1}{E_\tau} = \frac{1}{E(T)} + \frac{\partial}{\partial\sigma}\int_0^t K(T)e^{\beta(T)\sigma}dt, \tag{2.2}$$

where E(T) is the temperature-dependent modulus of elasticity. From (2.2) and (2.1) we obtain an equation from which we find the time and, consequently, the temperature at which a rod becomes unstable,

$$\frac{\pi^2}{\lambda^2\sigma} = \frac{1}{E(T)} + \int_0^t K(T)\beta(T)\exp[\beta(T)\sigma]\,dt. \tag{2.3}$$

For σ = 8.1 kG/mm^2 and θ = 0.52° C/sec we determined (by integration) the temperature of the loss of stability, T = 297° C. This temperature is indicated in Fig. 4 by vertical line S.

If the lateral disturbance is applied for a very short period of time so that the outer "fibers" of the material are slightly unloaded when the specimen is deflected, the tangent modulus E_T in (2.1) is replaced by the effective modulus E^*. Carrying out the calculations by the simplified method and replacing the actual specimen cross section by the ideal H-beam cross section in the estimation of the effective modulus for the core in a rod specimen which is here in the form

$$E^* = 2E(T)E_\tau / (E(T) + E_\tau), \tag{2.4}$$

we obtain (after substituting in (2.1) and carrying out simple transformations) an equation

$$\frac{\pi^2}{\lambda^2\sigma} = \frac{1}{E(T)} + \frac{1}{2}\int_0^t K(T)\beta(T)\exp[\beta(T)\sigma]\,dt, \tag{2.5}$$

from which the temperature corresponding to the moment of the loss of stability can be calculated. Numerical calculations for the same conditions give T_2 = 303° C which is indicated in Fig. 4 by line F.

If we apply a criterion based on the fact that the total strain at which a rod buckles in creep should be the same as that at which buckling takes place in the absence of creep [4], we obtain an equation

$$\epsilon_* = \epsilon + p = \pi^2 / \lambda^2. \tag{2.6}$$

The temperature strains, since they are independent of the force factors, are not taken into account. Having substituted the expression for the strains into (2.6), we derive the equation

$$\frac{\pi^2}{\lambda^2} = \frac{\sigma}{E(T)} + \int_0^t K(T)\exp[\beta(T)\sigma]dt \tag{2.7}$$

from which the temperature corresponding to the moment of the loss of stability can be determined. Numerical calculation using (2.7) gives (for the same conditions) T_3 = 313° C indicated in Fig. 4 by vertical line G.

It may be concluded on the basis of obtained results that all above described methods, though producing overestimated values, give a fairly true picture of the loss in stability of rods in creep at constant loads and monotonically increasing temperatures. The most accurate is the tangent-modulus method.

REFERENCES

1. O. V. Sosnin and N. G. Torshenov, "Compression and buckling in creep at monotonically increasing load," PMTF [Journal of Applied Mechanics and Technical Physics], no. 5, 1967.

2. R. L. Carlson and E. G. Bodine, "An experimental study of compressive creep: Behavior elevated temperatures," Proc. ASTM, vol. 57, 1957.

3. F. R. Stanley, Analysis of the Weight and Strength of Aircraft Structures [Russian translation], Oborongiz, 1957.

4. G. Gerard, "A creep-buckling hypothesis," J. Aeronaut, Sci., vol. 19, no. 10, 1952.

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